**United Kingdom
Mathematics Trust**

Intermediate Mathematical Challenge

Wednesday 2 and Thursday 3 February 2022

Organised by the United Kingdom Mathematics Trust

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Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   Each incorrect answer to Questions 16-20 loses 1 mark;
   Each incorrect answer to Questions 21-25 loses 2 marks.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

1. How many hours is 6 minutes?

     A   0.06   B   0.1   C   0.6   D   10   E   360

2. My recipe for apple crumble uses 100 g of flour, 50 g of butter and 50 g of sugar to make the crumble topping. When my family come for a meal, I have to use two and a half times each amount to provide enough crumble. In total, how much crumble topping do I then make?

     A   0.5 kg   B   2 kg   C   2.5 kg   D   5 kg   E   50 kg

3. In the Caribbean, loggerhead turtles lay three million eggs in twenty thousand nests. On average, how many eggs are in each nest?

     A   15   B   150   C   1500   D   15 000   E   150 000

4. Workers digging a tunnel for an underground railway complete 5 metres of tunnel on a typical day. Working every day, how long will it take them to dig a tunnel of length 2 kilometres?

     A   three months   B   six months   C   just under a year   D   just over a year
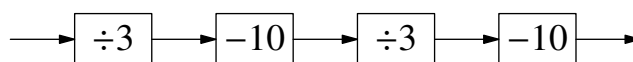   E   nearly two years

5. Which of the following has the same value as $10\,006 - 8008$?

     A $10\,007 - 8007$ B $100\,060 - 80\,080$ C $10\,000 - 8002$ D $106 - 88$
     E $5003 - 4004$

6. What is 20% of $3\frac{3}{4}$?

     A $\dfrac{123}{200}$ B $\dfrac{13}{20}$ C $\dfrac{7}{10}$ D $\dfrac{3}{4}$ E $\dfrac{4}{5}$

7. A function machine does the four operations shown in order.

     $\longrightarrow \boxed{\div 3} \longrightarrow \boxed{-10} \longrightarrow \boxed{\div 3} \longrightarrow \boxed{-10} \longrightarrow$

Iris inputs a positive integer and the output is also a positive integer.
What is the smallest possible number which Iris could have input?

     A   9   B   84   C   102   D   120   E   129

8. What is the difference between 40% of 50% of 60 and 50% of 60% of 70?

     A   9   B   8   C   7   D   6   E   5

9. A number x is greater than 2022. Which is the smallest of the following?

     A $\dfrac{x}{2022}$ B $\dfrac{2022}{x-1}$ C $\dfrac{x+1}{2022}$ D $\dfrac{2022}{x}$ E $\dfrac{2022}{x+1}$

10. One hundred rectangles are arranged edge-to-edge in a continuation of the pattern shown.

      . . .

Each rectangle measures 3 cm by 1 cm. What is the perimeter, in cm, of the completed shape?

     A   800   B   700   C   602   D   600   E   502

11. The Universal Magazine of Knowledge and Pleasure (Vol. 1, 1747) asked the following question. "What number is that, whose quarter shall be 9 more than the whole?" What is the correct answer?

     A   12   B   9   C   8   D -8 E -12

12. The shape shown is made up of three similar right-angled triangles. The smallest triangle has two sides of side-length 2, as shown.
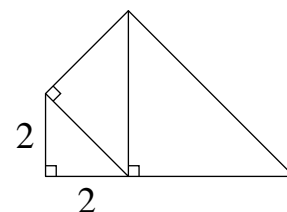
What is the area of the shape?

2

2

     A 14   B $12 + 12\sqrt{2}$ C 28   D $24 + 20\sqrt{2}$ E 56

13. How many sets of three consecutive integers are there in which the sum of the three integers equals their product ?

     A 0   B 2   C 3   D 4   E 5

14. In a number pyramid, each cell above the bottom row contains the sum of the numbers in the two cells immediately below it. The three numbers on the second row are all equal, and are all integers. Which of these statements *must* be true?

third row

second row
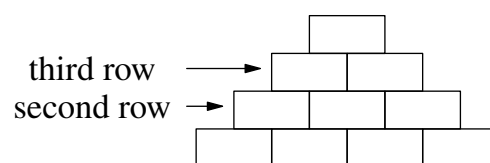
     A The bottom row contains at least one zero   B The third row contains at least one zero
     C The top number is a multiple of three   D The top number is a multiple of four
     E None of the above

15. Reflection in the line l transforms the point with coordinates $(5, 3)$ into the point with coordinates $(1, -1)$.
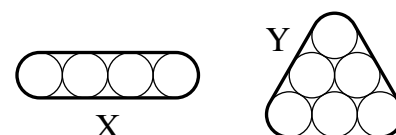What is the equation of the line l?

     A $y = x - 2$ B $y = 1$ C $x = 3$ D $y = 2 - x$ E $y = 4 - x$

16. What is half of 4^{2022}?

     A 4^{1011} B 2^{4044} C 4^{2021} D 2^{4043} E 2^{1011}

17. The first figure shows four touching circles of radius 1 cm in a horizontal row, held together tightly by an outer band X.

Y

X

The second figure shows six touching circles of radius 1 cm, again held tightly together by a surrounding band Y.
Which of the following statements is true?

     A X is 2 cm longer than Y   B X is 1 cm longer than Y
     C X and Y are the same length   D Y is 1 cm longer than X
     E Y is 2 cm longer than X

18. Dick Turnip sold his horse, Slack Bess, for £56. The percentage profit he made was numerically the same as the cost, in pounds, of his horse. What was the cost of his horse?

   A £36   B £40   C £45   D £48   E £50

19. A sector of a circle has radius 6 and arc length 10, as shown.

What is the area of the sector?
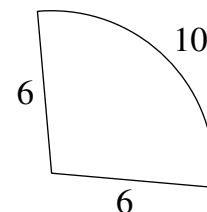
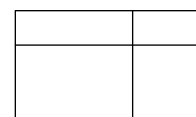   A 30   B 36   C 40   D 60   E 66

20. Aroon is asked to choose five integers so that the mode is 2 more than the median and the mean is 2 less than the median. What is the largest possible value of the range of Aroon's five integers?

    A 2   B 5   C 12   D 15

    E The largest possible range depends on the integers chosen

21. The diagram shows a shaded semicircle of diameter 4, from which a smaller semicircle has been removed. The two semicircles touch at exactly three points. What fraction of the larger semicircle is shaded?
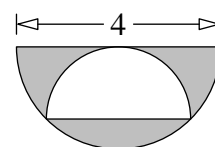
    A $\dfrac{2}{\pi}$ B $\dfrac{1}{2}$ C $\dfrac{\sqrt{2}}{3}$ D $\dfrac{\sqrt{2}}{2}$ E $\dfrac{3}{4\pi}$

22. A rectangle with integer side-lengths is divided into four smaller rectangles, as shown. The perimeters of the largest and smallest of these smaller rectangles are 28 cm and 12 cm.
Which of the following is a possible area of the original rectangle?

    A 90 cm^2 B 92 cm^2 C 94 cm^2 D 96 cm^2 E 98 cm^2

23. Two squares are drawn inside a regular hexagon with side-length 2, as shown. What is the area of the overlap of the two squares?
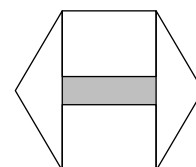
    A 2   B $2 - \sqrt{3}$ C $4 - \sqrt{3}$ D $4 - 2\sqrt{3}$ E $8 - 4\sqrt{3}$

24. Pete's pies all cost an integer number of pounds. A cherry pie costs the same as two apple pies. A blueberry pie costs the same as two damson pies. A cherry pie and two damson pies cost the same as an apple pie and two blueberry pies. Paul buys one of each type of pie.
Which of the following could be the amount he spends?

    A £16   B £18   C £20   D £22   E £24

25. Alvita is planning a garden patio to be made from identical square paving stones laid out in a rectangle measuring x stones by y stones. She finds that when she adds a border of width one stone around the patio, the area of the border is equal to the original area of the patio.
How many possible values for x are there?

    A 1   B 2   C 4   D 8   E 16