Singapore & Asian Schools Math Olympiad

Secondary 1 (Grade 7) – GEP Practice

2020

Contest Problems with Full Solutions

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Section A (Correct answer – 2 points | No answer – 0 points | Incorrect answer – minus 1 point)

Question 1

Find the value of the following.

$$2020 \times 2020 - 2019 \times 2021$$

A. 2020
B. 4040
C. 1
D. 2
E. None of the above

Question 2

Three apples and four oranges cost $10.90, while five apples and seven oranges cost $18.90. How much do nine apples and 13 oranges cost?

A. $20.60

B. $33.10

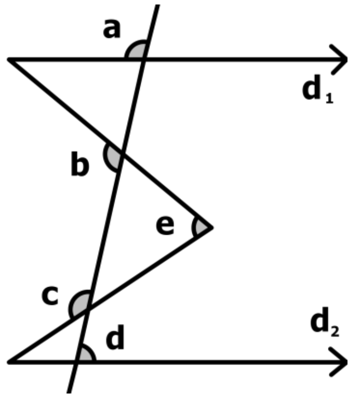
C. $34.20

D. $34.90

E. None of the above

Question 3

In the following diagram, d_1 is parallel to d_2 and $a + b + c + d = 460°$. What is angle e in degrees?



A. $100°$
B. $80°$
C. $70°$
D. $60°$
E. None of the above

Question 4

Find the next term of the following sequence.

1935, 1940, 1948, 1962, 1985, 2020, …

A. 2070
B. 2055
C. 2060
D. 2067
E. None of the above

Question 5

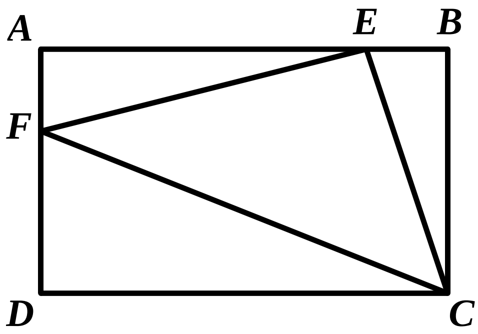
Find the last digit of the following product.

$$2^{2020} \times 3^{2022}$$

A. 6
B. 9
C. 3
D. 4
E. None of the above

Question 6

In the rectangle $ABCD$, point E is on the side AB while point F in on the side AD such that $BE = \frac{1}{4}AE$ and $DF = \frac{2}{3}AD$. What is the ratio of areas of the rectangle $ABCD$ to triangle FEC?



A. 30 : 13
B. 40 : 17
C. 48 : 23
D. 24 : 11
E. None of the above

Question 7

The six-digit number 2X475Y is divisible by 36. How many possible values of X are there?

A. 1
B. 2
C. 3
D. 4
E. None of the above

Question 8

There are roses and lilies at a flower shop. Four roses cost $7 while three lilies cost $8. Billy bought some roses and lilies and paid $86. Which of the following can be the number of roses he bought?

A. 16
B. 24
C. 44
D. 36
E. None of the above

Question 9

How many triples (x, y, z) of positive integers are there such that $x + y + z = 14$?

Take note that (x, y, z), (x, z, y), (y, z, x), (y, x, z), (z, x, y) and (z, y, x) are the same triple.

A. 12
B. 11
C. 13
D. 16
E. None of the above

Question 10

What is the least number of weights required to weigh any objects of integer number of grams from 1 to 35 grams? The weights must be put on one plate while the object is put on the other plate. Also, the weights must be in an integer number of grams.

A. 35
B. 10
C. 7
D. 6
E. None of the above

Question 11

Each face of a soccer ball is either a pentagon or a hexagon. Each pentagonal face is adjacent to five hexagonal faces and each hexagonal face is adjacent to three pentagonal and three hexagonal faces. If the ball has 12 pentagonal faces, how many hexagonal faces are there?

A. 12
B. 20
C. 24
D. 8
E. None of the above

Question 12

Alice, Jane, Philip and Victor were born in 1982, 1983, 1984, 1985 in the cities of Athens, Moscow, Paris and Singapore, though not in that order. It is given that

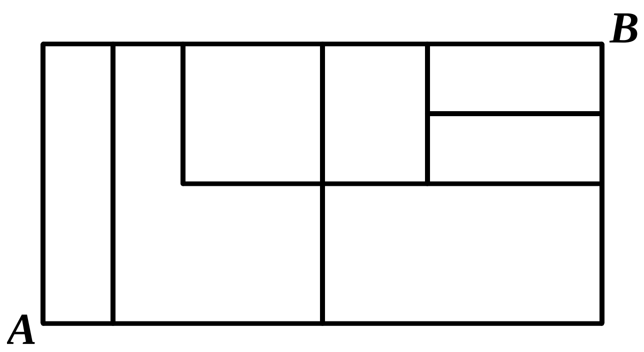
- The person born in Paris is one year older than the one born in Singapore.
- Victor was born one year later than the person born in Paris.
- Philip was born in Moscow.
- Alice was born two years later than Victor was.

Which year was Jane born in?

A. 1982
B. 1983
C. 1984
D. 1985
E. Impossible to determine

Question 13

In the diagram below, an ant can move only upwards or rightwards. How many ways are there for the ant to get from point A to point B along the existing lines?



A. 7
B. 12
C. 8
D. 2
E. None of the above

Question 14

Henry has a bag with 13 yellow, seven brown, 25 red and ten green balls. All the balls are of the same size and shape. What is the least number of balls Henry needs to take out without looking to make sure that he gets three different coloured balls?

A. 3
B. 20
C. 39
D. 48
E. None of the above

Question 15

How many four-digit numbers are there between 3700 and 9600 that can be formed using only the digits 3, 7, 5, 6, 0 or 9 without repetition of any digits?

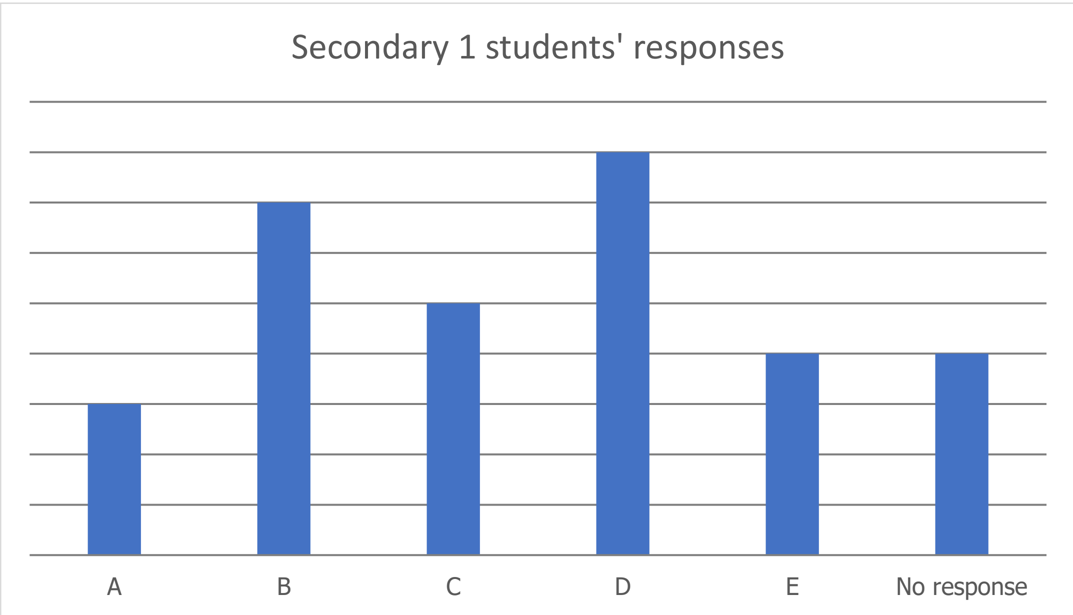
A. 300
B. 100
C. 264
D. 240
E. None of the above

Section B (Correct answer – 4 points | Incorrect or No answer – 0 points)

When an answer is a 1-digit number, shade "0" for the tens, hundreds and thousands place.
Example: if the answer is 7, then shade 0007
When an answer is a 2-digit number, shade "0" for the hundreds and thousands place.
Example: if the answer is 23, then shade 0023
When an answer is a 3-digit number, shade "0" for the thousands place.
Example: if the answer is 785, then shade 0785
When an answer is a 4-digit number, shade as it is.
Example: if the answer is 4196, then shade 4196

Question 16

The following bar chart shows the responses made by all Secondary 1 students from Integrity Secondary School in a multiple-choice question. All the horizontal lines are equally spaced. The correct answer scores 2 points, 0 points for no response and –1 point for the wrong answer. The total points scored by all the students is –120. If the correct answer is B, how many students solved the question correctly?



Question 17

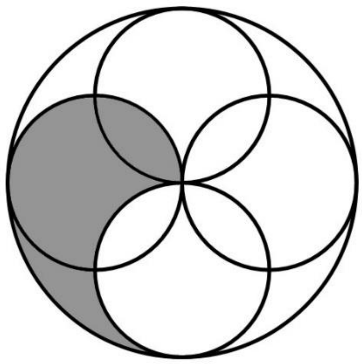
Andrea and Claire left from Woodlands and HarbourFront respectively and travelled towards each other at the same time. When Andrea arrived at HarbourFront, Claire needed to travel for another 5 km more to reach Woodlands. If Andrea travelled 20% faster than Claire, find the distance, in km, between Woodlands and HarbourFront.

Question 18

Find the smallest positive integer n for which $3993n$ is a multiple of 2475.

Question 19

In the diagram, four identical circles touch the large circle and pass through the centre of the large circle. If the diameter of the large circle is 28 cm, find the area (in cm^2) of the shaded region. (Use $\pi = \frac{22}{7}$)



Question 20

Denote $\{n\} = 1 + 3 + 5 + \cdots + (2n - 1)$. For example, $\{5\} = 1 + 3 + 5 + 7 + (2 \times 5 - 1)$. Given that $\frac{\{m\}}{m} + m = 2020$, what is the value of m?

Question 21

What is the smallest positive integer that has exactly 3 odd divisors and 3 even divisors?

Question 22

The operator $\wedge$ acts on two integers to give the following outcomes:

$$4\wedge 8 = 2412$$

$$5\wedge 9 = 2812$$

$$1\wedge 7 = 1618$$

$$2\wedge 3 = 103$$

What is the value of $6\wedge 7$?

Question 23

Peter has five different books of different subjects to be placed on a single-decked shelf. He does not want to place the Physics book next to the Biology one. In how many ways can he place all his books?

Question 24

Peter and Frank together can build a house in 12 days. Frank and George together can build the same house in six days. It is given that each of them works exactly nine hours per day. If Peter and George together can build the house in 6.5 days, how long (in hours) will Peter, Frank and George work together to build the house?

Round your answer to the nearest hour.

Question 25

In the following cryptarithm, all the different letters stand for different digits.

			W	I	N
+	S	A	S	M	O
	M	E	D	A	L

If N = 7 and M = 6, find the value of the sum S + A + S + M + O.

END OF PAPER

Solutions to SASMO 2020 Secondary 1 (Grade 7)

Question 1

Let $x = 2020$, then $2020 \times 2020 - 2019 \times 2021 = x^2 - (x-1)(x+1) = x^2 - (x^2 - 1) = \mathbf{1}$

<u>Answer:</u> **(C)**

Question 2

Let the cost of each apple be x dollars, and let the cost of each orange be y dollars.

Then, $3x + 4y = 10.90$ (1) and $5x + 7y = 18.90$ (2)

Subtract equation (1) from equation (2) to get $2x + 3y = 8.00$ (3)

Finally, add equation (2) to 2 times of equation (3) to get $9x + 13y = 5x + 7y + 2(2x + 3y) = 18.90 + 8.00 \times 2 = \$\mathbf{34.90}$

<u>Answer:</u> **(D)**

Question 3

Since lines d_1 and d_2 are parallel, $a + d = 180°$ This means $b + c = 460° - 180° = 280°$

Finally, observe that angles $e, (180 - b)°$, and $(180 - c)°$ form a triangle, or

$$e + 180° - b + 180° - c = 180°$$

$$e + 180° = b + c = 280°$$

$$e = \mathbf{100}°$$

Question 4

Find the difference between 2 adjacent terms of the pattern above (e.g. 2nd term – 1st term), then list down the 5 differences to form a new sequence, which is:

$$5, 8, 14, 23, 35$$

Now the pattern is clear, as the differences between 2 adjacent terms in the new sequence are increasing multiples of 3, hence the next term of the new sequence will be $35 + 3 \times 5 = 50$, and the next term of the sequence in question will be $2020 + 50 = \mathbf{2070}$

Answer: **(A)**

Question 5

$$2^{2020} \times 3^{2022} = (2 \times 3)^{2020} \times 3^2 = 6^{2020} \times 3^2$$

Note that the last digit of 6^n will always be 6 if n is a positive integer. Hence, the last digit of $6^{2020} \times 3^2$ is equivalent to the last digit of 6×3^2, which is **4.**

Answer: **(D)**

Question 6

Let AB = DC = $5x$, then AE = $4x$ and BE = x. Let AD = BC = $3y$, then AF = y and FD = $2y$.

$$Area(ABCD) = 15xy$$

$$Area(AEF) = \frac{1}{2} \times 4x \times y = 2xy$$

$$Area(FCD) = \frac{1}{2} \times 2y \times 5x = 5xy$$

$$Area(EBC) = \frac{1}{2} \times x \times 3y = \frac{3xy}{2}$$

And $Area(FEC) = 15xy - 2xy - 5xy - \frac{3xy}{2} = \frac{13xy}{2}$

Hence $Area(ABCD) : Area(FEC) = 15 : \frac{13}{2}$ = **30 : 13**

Answer: **(A)**

Question 7

The number 2X475Y must be divisible by 9 and 4 for it to be divisible by 36.

Hence the last 2 digits of the number, or 5Y, must be divisible by 4. This means $Y = 2\ or\ 6$.

If $Y = 2,$ the sum of $2 + X + 4 + 7 + 5 + 2$ must be divisible by 9, and $X = 7$ is the only possible choice. If $Y = 6,$ the sum of $2 + X + 4 + 7 + 5 + 6$ must be divisible by 9, and $X = 3$ is the only possible choice.

Hence, there are **2** possible values of X.

Answer: **(B)**

Question 8

To get $86, the number of roses must be divisible by 4 and the number of lilies must be divisible by 3. Let the number of roses be $4 \times a$ and the number of lilies be $3 \times b$. Hence

$$7 \times a + 8 \times b = 86$$

When $a = 1,$ b is not a whole number.

When $a = 2,$ b is 9.

Checking other values of a from 3 to 12, the possible value of a is 10. Then $b = 9$ and 2 or $3 \times b = 27$ and 6. Thus, none of the options A to D are possible and the answer is **option E**.

Answer: **(E)**

Question 9

List down the triples by the largest number among (x, y, z).

For example, if the largest number is 12, then there is only one possible triple (12, 1, 1).

Largest number	Possible triples	Largest number	Possible triples
12	12, 1, 1	8	8, 3, 3
11	11, 2, 1	7	7, 6, 1
10	10, 3, 1		7, 5, 2
	10, 2, 2		7, 4, 3
9	9, 4, 1	6	6, 6, 2
	9, 3, 2		6, 5, 3
8	8, 5, 1		6, 4, 4
	8, 4, 2	5	5, 5, 4

There are **16** such triples.

Answer: **(D)**

Question 10

To weigh 1-gram objects, we need 1-gram weights.

We can weigh any objects of integer number of grams from 1 to 35 grams using thirty-five 1-gram weights. However, that is not the least number of weights.

Let us optimise to get the least number of weights.

Substitute 18 of 35 one-gram weights to one 18 g weight. So now we have one 18-gram weight and 17 one-gram weights. We still can weigh any weight from 1 to 35 grams.

Next, substitute 9 of 17 one-gram weights to one 9-gram weight. Continue the same process until we have 18 g, 9 g, 4 g, 2 g, 1 g and 1 g weights. We can weigh any weight from 1 to 35 grams using these **6 weights**.

Using mathematics Induction, it can be proved that $[\log_2 N] + 1$ is the least number of weights required to measure N objects with respective weights from 1 to N, where the weights are $\left\lceil \frac{N}{2} \right\rceil, \left\lceil \frac{N}{4} \right\rceil, \left\lceil \frac{N}{8} \right\rceil, \dots, 1$ and 1.

Answer: **(D)**

Question 11

Each pentagon has 5 sides and each hexagon has 6 sides.

Total number of sides of the pentagons $= 5 \times 12 = 60$

Total number of hexagons= $\frac{60}{3} = \mathbf{20}$, since each hexagonal face is adjacent to 3 hexagonal faces.

Answer: **(B)**

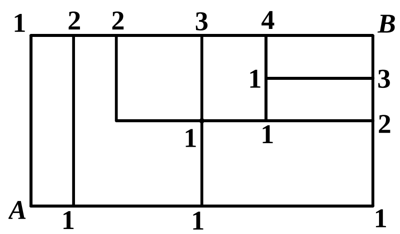
Question 12

Consider the 4 statements above. From statements 1, 2 and 4, it is clear that Victor is the 2nd oldest (born in 1983), and Alice is the youngest (born in 1985). Furthermore, Victor is born in Singapore, Philip is born in Moscow, and Alice cannot be born in Paris as she is the youngest (while the person born in Paris is the oldest). This leaves Jane to be born in Paris. Since she is the oldest, then she must be born in **1982**.

Answer: **(A)**

Question 13

There is only 1 way from point A to the top-left point. Thus, we label that point with '1'. Next, there are 1+1=2 ways to the second left point on top and we label it with '2'. We label the remaining points on the figure similarly as shown below.



From the figure, the number of ways from A to B is 4 + 3 = **7.**

Answer: **(A)**

Question 14

For such questions, it is always best to consider the "worst-case" scenario to determine the least number of balls which needs to be taken out.

Henry takes out 25 balls, unfortunately all 25 balls turn out to be red. Next, Henry takes out 13 more balls, but all of them turn out to be yellow. But after taking out 38 balls, Henry cannot be so "unlucky" anymore, hence the **39th** ball taken out must be either brown or green, making it 3 different coloured balls.

Answer: **(C)**

Question 15

We consider 3 cases.

Case 1: The thousands digit is 3. Then there are 2 choices (7 or 9) for the hundreds digit, 4 choices for the tens digit (0,5,6 or 7/9), depending on the choice of the hundreds digit, and 3 choices for the ones digit, since one of the 4 numbers above will be chosen for the tens digit and digits cannot be repeated.

Total number of choices = $2 \times 4 \times 3 = 24$

Case 2: The thousands digit is 5,6 or 7 (3 choices). Then there are 5 choices (out of the numbers 3,7,5,6,0,9 but one of them will be used for the thousands place) for the hundreds digit, 4 choices for the tens digit, and 3 choices for the ones digit.

Total number of choices = $3 \times 5 \times 4 \times 3 = 180$

Case 3: The thousands digit is 9. Then there are 3 choices (0,3,5) for the hundreds digit, 4 choices for the tens digit (with a similar reasoning to cases 1 and 2), and 3 choices for the ones digit.

Total number of choices = $3 \times 4 \times 3 = 36$

Summing up the 3 cases, we get a total of **240** numbers which fulfil the question's criteria.

Answer: **(D)**

Question 16

Let each space of the bar graph be x students.

Number of students who answered the question correctly = number of students who chose B = $7x$ Hence, a total of $14x$ points were scored from them.

Number of students who answered the question wrongly = number of students who chose A, C, D or E = $3x + 5x + 8x + 4x = 20x$. A total of $-20x$ points were scored from them.

Total number of points = $14x - 20x = -6x = -120$, or $x = 20$.

Number of students who answered the question correctly = $7x = \mathbf{140}$

Answer: **140**

Question 17

Let the speed of Andrea be $6x$ km/h and speed of Claire be $5x$ km/h

Time taken for Andrea to reach Harbourfront = time taken for Claire to reach 5 km away from Woodlands = $\frac{D}{6x}$, D km is the distance between Woodlands and Harbourfront.

Distance Claire travelled = speed of Claire × time, or $D - 5 = \frac{D}{6x} \times 5x$ which gives $D - 5 = \frac{5}{6}D$

$$D = \mathbf{30}$$

Answer: **30**

Question 18

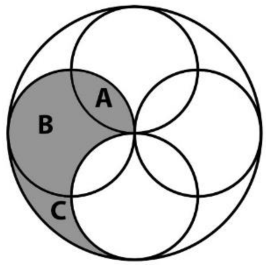
First, we factorise 3993 and 2475 to get $3993 = 3 \times 11^3$ and $2475 = 5^2 \times 3^2 \times 11$.

It is given that $\frac{3993n}{2475} = \frac{3\times11^3\times n}{5^2\times3^2\times11} = \frac{11^2\times n}{5^2\times3} = \frac{11^2\times n}{75}$ is a positive integer. Hence n is divisible by 75 and the smallest possible value of n is 75.

Answer: **75**

Question 19

Denote three areas A, B and C in the shaded region as shown.



We can notice that the area of the large circle consists of 4 parts of each A, B and C. Hence

$$4A + 4B + 4C = \pi r^2 = \frac{22}{7} \times \left(\frac{28}{2}\right)^2 = 616$$

and the shaded regions is

$$A + B + C = 616 \div 4 = \mathbf{154}.$$

Answer: **154**

Question 20

We attempt to correlate $\{n\}$ with n by finding a pattern.

Observe $\{1\} = 1 = 1^2$, $\{2\} = 1 + 3 = 4 = 2^2$, $\{3\} = 1 + 3 + 5 = 9 = 3^2$, $\{4\} = 1 + 3 + 5 + (2 \times 4 - 1) = 16 = 4^2$

Hence, $\{m\} = m^2$ Now, $\frac{\{m\}}{m} + m = m + m$ (m is not 0) $= 2m = 2020$

$m =$ **1010**

Answer: **1010**

Question 21

Let this integer be n.

Since an even number divides n, n must be even. We can then assume that the remaining 2 even divisors of n (besides n itself) are 2 and $2m$, where m is an odd prime.

The smallest possible value of m is 3, so when expressed as a product of 2 numbers, $n = 1 \times n, 2 \times k$ OR $2 \times 2m = 4m$, and $m \times k$ OR $m \times 2m = 2m^2$. k is an arbitrary positive integer, and another ODD divisor of n.

Clearly, $n = 4m = 12$ does not satisfy the question's criteria. On the other hand, $n = 2m^2 = \mathbf{18}$ has 3 odd factors and 3 even factors.

Answer: **18**

Question 22

The pattern is as follows: $a\wedge b = [2(a+b)][3(b-a)]$

where we don't multiply the square brackets, but write these numbers next to each other. E.g. $4\wedge 8 = [2(8+4)][3(8-4)] = 2412$

This is a very common "trick" in SASMO, and must be known by heart!

$6\wedge 7 = [2(6+7)][3(7-6)] = \mathbf{263}$

<u>Answer:</u> **263**

Question 23

Total number of ways to place the 5 books without restraint $= 5! = 120$

We now want to find the number of ways where the Physics and Biology books must be placed together. Treat the adjacent physics and biology books as 1 "multi-book", and this multi-book can be arranged in 2 different ways (by swapping the positions of the Physics and Biology books).

This "multi-book" can be arranged with the 3 other books in a total of $2 \times 4! = 48$ ways.

Total number of ways not to place the physics book next to the biology book is $120 - 48 =$**72**

<u>Answer:</u> **72**

Question 24

Let Peter, Frank and George be able to build $\frac{1}{p}, \frac{1}{F}$ and $\frac{1}{G}$ of a house in 9 hours, or one work-day. Then

$$\frac{1}{P} + \frac{1}{F} = \frac{1}{12}, \frac{1}{F} + \frac{1}{G} = \frac{1}{6}, \frac{1}{P} + \frac{1}{G} = \frac{1}{6.5} = \frac{2}{13}$$

$\frac{1}{P} + \frac{1}{F} + \frac{1}{G}$ can build $\frac{\frac{1}{12}+\frac{1}{6}+\frac{2}{13}}{2} = \frac{21}{104}$ of a house in 9 hours.

Number of hours these 3 people must work together to build a house= $\frac{9}{(\frac{21}{104})} = \frac{312}{7} =$ $\mathbf{45}$ (to the nearest hour)

<u>Answer:</u> **45**

Question 25

S = 5 because S≠M and there can be a carrying over of maximum 1 to S, so S + 1 = 6. A = 9 because A must carry a 1 over to S, so A + 1 ≥ 10. When A = 9, E = 0.

Substitute N = 7, M = 6, S = 5, A = 9 and E = 0 to get the following:

			W	I	7
+	5	9	5	6	O
	6	0	D	9	L

We are now left with 5 numbers which W, I, O, D and L represent, namely 1,2,3,4 and 8. With these numbers, there are only 2 cases left to consider for 7 + O = L.

Case 1: O = 1 and L = 8. Then I = 3 for I + 6 = 9. That leaves W + 5 = D + 10 (remember that D needs to carry a 1 over to A) and (W,D) = (2,4) or (4,2). This is impossible.

Case 2: O = 4 and L = 1. Then I = 2 for 1 + I + 6 = 9. Finally, W = 8 and D = 3 for W + 5 = D + 10

S + A + S + M + O = 5 + 9 + 5 + 6 + 4 = **29**

<u>Answer:</u> **29**