



## **Secondary 2 (Grade 8) – GEP Practice**

# **2020**

## **Contest Problems with Full Solutions**

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**Section A** (Correct answer – 2 points | No answer – 0 points | Incorrect answer – minus 1 point)**Question 1**

Find the value of the following.

$$2020 \times 20 - 20 \times 20 + 2020$$

- A. 2020
- B. 42020
- C. 40000
- D. 40202
- E. None of the above

**Question 2**Simplify  $3^1 \times 3^3 \times 3^5 \times 3^7 \times 3^9 \times 3^{11} \times 3^{13} \times 3^{15} \times 3^{17} \times \left(\frac{1}{3}\right)^{75}$ .

- A. 27
- B. 81
- C. 243
- D. 729
- E. None of the above

**Question 3**Which of the following is equal to  $ax - 2by + 3c$  if  $x = 3a - 4b^2$ ,  $y = 4b - 3c^2$ , and  $z = -2bc$ ?

- A.  $3a^2 - 4ab^2 - 8b^2$
- B.  $3a^2 - 4ab^2 - 8b^2 + 12bc^2$
- C.  $3a^2 - 4ab^2 - 8b^2 - 12bc^2$
- D.  $3a^2 - 4ab^2 + 8b^2 + 12bc^2$
- E. None of the above

**Question 4**

Find the value of  $N$  in the equation below.

$$\frac{1 + 2 + 3 + \cdots + 120}{110} = \frac{2 + 4 + 6 + N}{2}$$

- A. 232
- B. 242
- C. 252
- D. 262
- E. None of the above

**Question 5**

A large rectangle in the figure below is divided into nine smaller rectangles. The areas of five rectangles have been given. What is the sum of the areas of the remaining four rectangles?

	<b>1</b>	
$\frac{1}{6}$		$\frac{1}{2}$
$\frac{1}{3}$	$\frac{2}{3}$	

- A.  $\frac{10}{3}$
- B. 2
- C. 3
- D.  $\frac{5}{2}$
- E. None of the above

**Question 6**

Denote  $n\Delta m = (n-1)(n-2)\cdots(n-m)$ . For example,  $5\Delta 3 = 4 \times 3 \times 2$ . If  $8\Delta x = 840$ , what is the value of  $x\Delta(x-3)$ ?

- A. 6
- B. 12
- C. 24
- D. 36
- E. None of the above

**Question 7**

The value of  $\frac{2022^3 - 2 \times 2022^2 - 2020}{2021^3 + 2 \times 2021^2 - 2023}$  can be written as  $\frac{a}{b}$  where  $HCF(a, b) = 1$ . What is the value of  $a + b$ ?

- A. 4039
- B. 4041
- C. 4043
- D. 4045
- E. None of the above

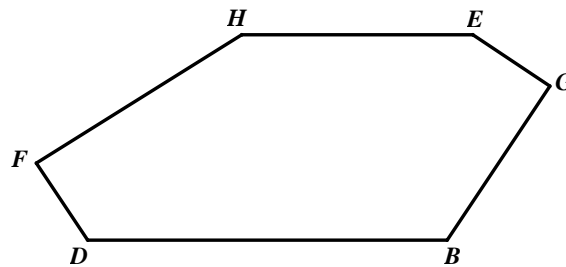
**Question 8**

If  $x < -4$ , simplify the expression  $\left|1 - \sqrt{(x+3)^2}\right|$ .

- A.  $x + 2$
- B.  $-x - 2$
- C.  $x + 4$
- D.  $-x - 4$
- E. None of the above

**Question 9**

In the figure below,  $HE$  is parallel to  $DB$ ,  $GE$  is perpendicular to  $BG$ , and  $DF$  is also perpendicular to  $HF$ . If  $\angle H = 138^\circ$ , and the measurement of  $\angle B$  is 4 degrees more than the measurement of  $\angle D$ , find the measurement of  $\angle E$ .



- A.  $134^\circ$
- B.  $44^\circ$
- C.  $46^\circ$
- D.  $138^\circ$
- E. None of the above

**Question 10**

In the list below, how many fractions are in its simplest form?

$$\frac{1}{2020}, \frac{2}{2020}, \frac{3}{2020}, \dots, \frac{199}{2020}, \frac{200}{2020}$$

- A. 121
- B. 59
- C. 79
- D. 144
- E. None of the above

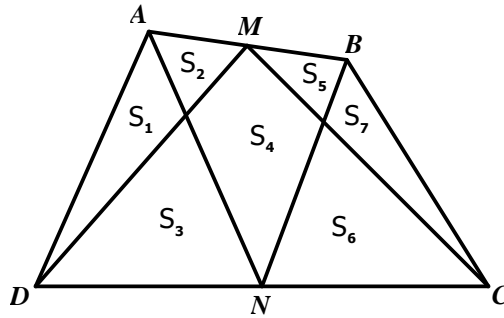
**Question 11**

Boris wants to write all five-digit numbers using all the digits 1, 2, 3, 4 and 5. The numbers are such that the even digits are in increasing order from left to right, for example 32145 and 25341. How many numbers can Boris write?

- A. 45
- B. 52
- C. 60
- D. 64
- E. None of the above

**Question 12**

In quadrilateral  $ABCD$ ,  $M$  and  $N$  are midpoints of sides  $AB$  and  $DC$ , respectively. The segments  $AN$ ,  $BN$ ,  $DM$  and  $CM$  divide the quadrilateral into 7 parts  $S_1, S_2, S_3, S_4, S_5, S_6$  and  $S_7$  of different areas. Which of the following relationships involving the areas is true?



- A.  $S_2 + S_4 = S_6$
- B.  $S_1 + S_7 = S_4$
- C.  $S_2 + S_3 = S_4$
- D.  $S_1 + S_6 = S_4$
- E. None of the above

**Question 13**

If today is Tuesday, what day of the week will it be  $2^{2020}$  days from today?

- A. Wednesday
- B. Thursday
- C. Friday
- D. Saturday
- E. None of the above

**Question 14**

Ellie, Violet, Audrey, Tom and Chris are Secondary 1 or 3 students. They study in either Greenwich Secondary School or Westwood Secondary School. It is also given that:

- Tom and Chris are from different schools.
- Ellie and Audrey go to the same school.
- Three students go to Greenwich Secondary School and the other two are from Westwood Secondary School.
- Violet and Chris are from the same grade.
- Audrey and Tom study at different levels.
- Three students study in Secondary 1 and the other two students in Secondary 3.

If one of them is Secondary 3 student from Westwood Secondary School, who is that person?

- A. Ellie
- B. Violet
- C. Audrey
- D. Tom
- E. Chris

**Question 15**

In Mathematics, the product of the first  $n$  positive integers is written as  $n! = n \times (n - 1) \times \dots \times 1$ . For example,  $2! = 2 \times 1$  and  $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ . How many integers from 1 to 40 cannot be the number of consecutive digit "0"s at the end of  $n!$ , for some integer  $n$ ?

- A. 0
- B. 6
- C. 7
- D. 10
- E. None of the above



**Section B (Correct answer – 4 points | Incorrect or No answer – 0 points)**

When an answer is a 1-digit number, shade "0" for the tens, hundreds and thousands place.

*Example: if the answer is 7, then shade 0007*

When an answer is a 2-digit number, shade "0" for the hundreds and thousands place.

*Example: if the answer is 23, then shade 0023*

When an answer is a 3-digit number, shade "0" for the thousands place.

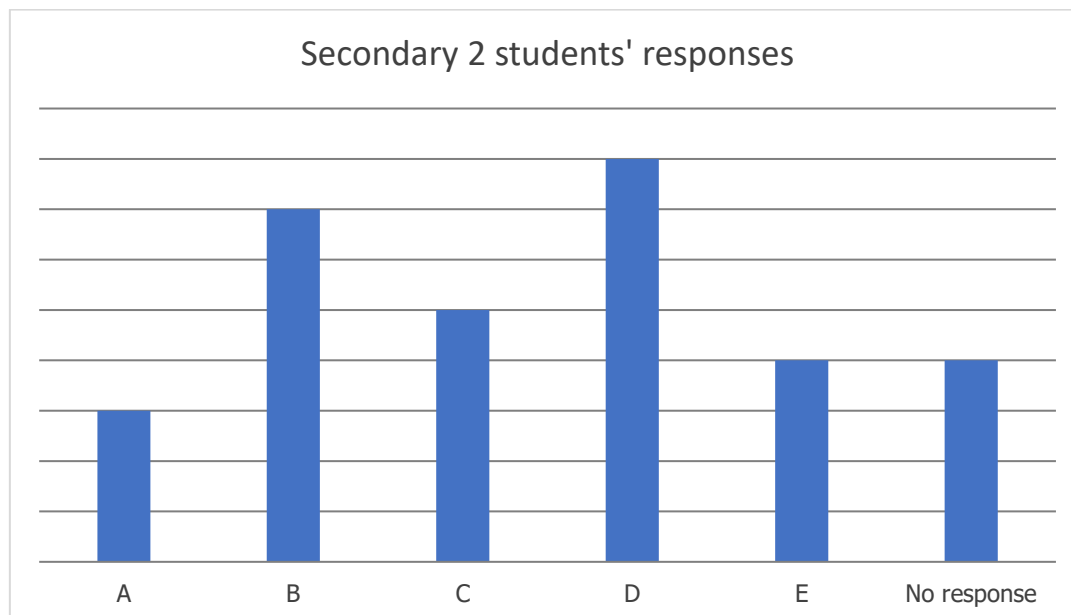
*Example: if the answer is 785, then shade 0785*

When an answer is a 4-digit number, shade as it is.

*Example: if the answer is 4196, then shade 4196*

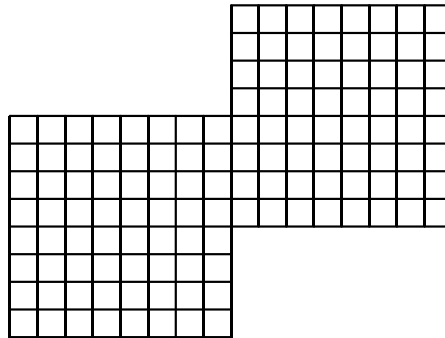
**Question 16**

The following bar chart shows the responses made by all Secondary 2 students from Integrity Secondary School in a multiple-choice question. All the horizontal lines are equally spaced. The correct answer scores 2 points, 0 points for no response and  $-1$  point for the wrong answer. The total points scored by all the students is  $-240$ . If the correct answer is B, how many students solved the question correctly?



**Question 17**

How many squares are there in the figure below?

**Question 18**

Chloe and her cousin left home for the cinema at 11.40 a.m. Chloe walked at 100 metres/minute while her cousin walked at 60 m/min. Ten minutes after arriving at the cinema, Chloe realised she had forgot her wallet and so she walked back home. On her way home, Chloe met her cousin who was still on the way to cinema, at 12.10 p.m. How far away was Chloe's home from the cinema, in metres?

**Question 19**

Find the sum of all positive integers  $n$  so that  $n^2 + 11n + 52$  is a perfect square.

**Question 20**

The six-digit number  $7A34B4$  is divisible by 36. If  $B$  is a prime number, find the value of  $A$ .

**Question 21**

The operator  $\wedge$  acts on two integers to give the following outcomes:

$$4 \wedge 8 = 2412$$

$$5 \wedge 9 = 2812$$

$$1 \wedge 7 = 1618$$

$$2 \wedge 3 = 103$$

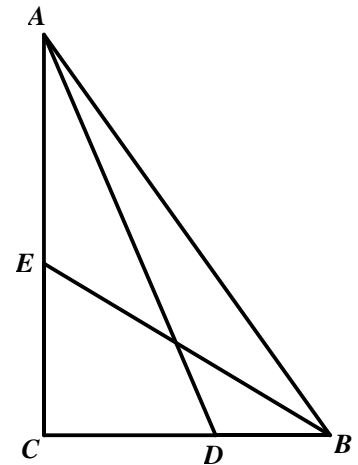
What is the value of  $7 \wedge 8$ ?

**Question 22**

An election for the mayoralty post consists of 5 candidates: A, B, C, D, and E. There are 1400 eligible voters. In the middle of the manual count, A got 300 votes, B got 154 votes, C got 194 votes, D got 281 votes, and E got 266 votes. There were also 9 invalid votes. What is the least number of votes that A needs to get in order to ensure that he wins the mayoralty post?

**Question 23**

In the diagram,  $ABC$  is a right-angled triangle with  $\angle C = 90^\circ$ . Points  $D$  and  $E$  are on the segments  $BC$  and  $AC$  such that  $BD:DC = 2:3$ ,  $AE:EC = 3:2$ ,  $AD = 4$ , and  $BE = \sqrt{10}$ . Find the value of  $57 \times AB^2$ .



**Question 24**

Two-way roads are constructed between Town A, Town B and Town C. It is given that the number of direct roads between any two of the towns is at least three and at most ten. One can go from Town A to Town C directly or via Town B in a total of 33 different ways. One can go from Town B to Town C directly or via Town A in a total of 23 different ways. In how many ways can one go from Town B to Town A?

**Question 25**

In the following cryptarithm, all the different letters stand for different digits.

$$\begin{array}{rcccccc} & & & W & I & N \\ + & S & A & S & M & O \\ \hline & M & E & D & A & L \\ \hline \end{array}$$

If  $N = 4$  and  $M = 6$ , find the value of the sum  $M + E + D + A + L$ .

**END OF PAPER**

## Solutions to SASMO 2020 Secondary 2 (Grade 8)

### Question 1

$$2020 \times 20 - 20 \times 20 + 2020 = 20(2020 - 20) + 2020 = 20 \times 2000 + 2020 = 40000 + 2020 = \mathbf{42020}$$

Answer: **(B)**

### Question 2

$$3^1 \times 3^3 \times 3^5 \times 3^7 \times 3^9 \times 3^{11} \times 3^{13} \times 3^{15} \times 3^{17} \times \left(\frac{1}{3}\right)^{75} = 3^{1+3+5+7+\dots+15+17} \times \left(\frac{1}{3}\right)^{75} = 3^{81} \times \left(\frac{1}{3}\right)^{75} = 3^6 = \mathbf{729}$$

Answer: **(D)**

### Question 3

$$ax - 2by + 3c = a(3a - 4b^2) - 2b(4b - 3c^2) + 3c = 3a^2 - 4ab^2 - 8b^2 - (-6bc^2) + 3c = 3a^2 - 4ab^2 - 8b^2 + 6bc^2 + 3c$$

None of the options in the question correspond to the above expression. Note that in this question, the value of  $z$  is redundant information.

Answer: **(E)**

### Question 4

$$\frac{1 + 2 + 3 + \dots + 120}{110} = \frac{\left(\frac{120 \times 121}{2}\right)}{110} = \frac{12 \times 10 \times 11 \times 11}{2 \times 10 \times 11} = 66 = \frac{12 + N}{2}$$

$$N = 66 \times 2 - 12 = \mathbf{120}$$

Answer: **(E)**

**Question 5**

The length of each side of the large rectangle is represented by A, B, C, D, E and F respectively, as shown in the diagram below.

	A	B	C
D	<b>W</b>	<b>1</b>	<b>X</b>
E	$\frac{1}{6}$	<b>Y</b>	$\frac{1}{2}$
F	$\frac{1}{3}$	$\frac{2}{3}$	<b>Z</b>

Observe that  $AE = \frac{1}{6}$  and  $AF = \frac{1}{3}$ , or  $F = 2E$ . Hence, area of  $Y = BE = \frac{1}{2} \times BF = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$  and area of  $Z = CF = 2 \times CE = 2 \times \frac{1}{2} = 1$ .

Observe that  $AF = \frac{1}{3}$ , and  $BF = \frac{2}{3}$ , or  $B = 2A$ . Hence, area of  $W = AD = \frac{1}{2} \times BD = \frac{1}{2} \times 1 = \frac{1}{2}$ .

Finally, observe that  $AE = \frac{1}{6}$  and  $CE = \frac{1}{2}$ , or  $C = 3A$ . Hence, area of  $X = CD = 3 \times AD = 3 \times \frac{1}{2} = \frac{3}{2}$ .

Sum of area of remaining 4 rectangles  $= \frac{1}{3} + 1 + \frac{1}{2} + \frac{3}{2} = \frac{10}{3}$

Answer: **(A)**

**Question 6**

$$8\Delta x = (8-1)(8-2) \dots (8-x) = 7 \times 6 \times \dots \times (8-x) = 840$$

Note that  $840 = 7 \times 120 = 7 \times 6 \times 20 = 7 \times 6 \times 5 \times 4$  so  $x = 8 - 4 = 4$

$$x\Delta(x-3) = 4\Delta(4-3) = 4 - 1 = \mathbf{3}$$

Answer: **(E)**

**Question 7**

Let  $n = 2021$

$$\begin{aligned}\text{Then } \frac{2022^3 - 2 \times 2022^2 - 2020}{2021^3 + 2 \times 2021^2 - 2023} &= \frac{(n+1)^3 - 2(n+1)^2 - n+1}{n^3 + 2n^2 - n - 2} = \frac{n^3 + 3n^2 + 3n + 1 - 2n^2 - 4n - 2 - n + 1}{(n-1)(n^2 + 3n + 2)} = \\ \frac{n^3 + n^2 - 2n}{(n-1)(n+1)(n+2)} &= \frac{n(n-1)(n+2)}{(n-1)(n+1)(n+2)} = \frac{n}{n+1} = \frac{2021}{2022}\end{aligned}$$

$$a + b = \mathbf{4043}$$

Answer: (C)

**Question 8**

If  $x < -4$ , then  $|x + 3| = -x - 3$

$$\left| 1 - \sqrt{(x+3)^2} \right| = |1 - |x+3|| = |1 - (-x-3)| = |x+4| = -x-4 \text{ (since } x < -4)$$

Answer: (D)

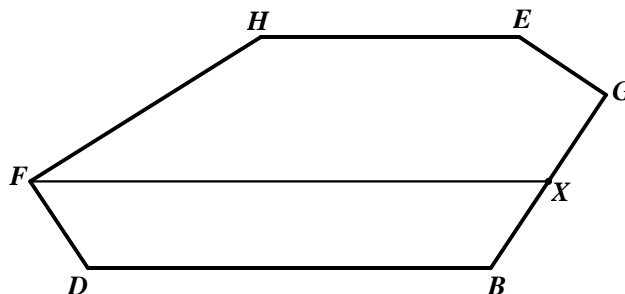
**Question 9**

The sum of the interior angles in a  $n$ -sided figure (or polygon) is  $(n-2) \times 180^\circ$ .

Draw line  $FX$ , which is parallel to lines  $HE$  and  $DB$ .

Then  $\angle DFX = 90^\circ - (180 - 138)^\circ = 48^\circ$  This means that  $\angle D = 180^\circ - \angle DFX = 180^\circ - 48^\circ = 132^\circ$  and  $\angle B = 132^\circ + 4 = 136^\circ$

Thus,  $\angle E = (6-2) \times 180^\circ - \angle G - \angle B - \angle D - \angle F - \angle H = 720^\circ - 90^\circ - 136^\circ - 132^\circ - 90^\circ - 138^\circ = \mathbf{134^\circ}$



Answer: (A)



**Question 10**

Since  $2020 = 2^2 \times 5 \times 101$ , any fraction which numerator is a multiple of 2, 5 OR 101 will not be in its simplest form.

From 1 to 200, only 1 multiple of 101, or 101, exists. Hence there is only 1 fraction whose numerator is a multiple of 101.

From 1 to 200, there exists  $\frac{200}{2} = 100$  multiples of 2,  $\frac{200}{5} = 40$  multiples of 5, and  $\frac{200}{10} = 20$  multiples of 10. Number of fractions which numerator is a multiple of 2 OR 5 =  $100 + 40 - 20 = 120$

Number of fractions not in their simplest form =  $120 + 1 = 121$ , this means that  $200 - 121 = 79$  fractions are in their simplest form.

Answer: (C)

**Question 11**

The implicit condition of the question is that any 5-digit numbers using all digits 1,2,3,4 and 5 (this will make a total of  $5! = 120$  such numbers without restrictions) can be written, so long as the digit 2 comes before digit 4.

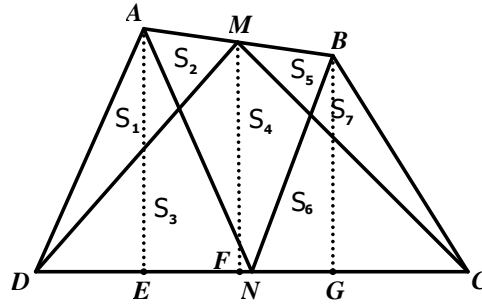
Notice that for every 1 number which digit 2 comes before digit 4, there will be one "counterpart" number which digit 4 comes before digit 2, while retaining the same position of its odd digits. For example, for 32145, its counterpart would be 34125.

This means that the number of numbers whose digit 2 comes before digit 4 = number of numbers whose digit 4 comes before digit 2, so Boris can write  $\frac{120}{2} = \mathbf{60}$  numbers.

Answer: (C)

**Question 12**

From  $A$ ,  $M$ , and  $B$ , draw 3 lines perpendicular to  $DC$  and label them as  $AE = h_a$ ,  $MF = h_m$ , and  $BG = h_b$  respectively.



Since  $M$  is the midpoint of  $AB$ ,  $MF$  is a midline. We have  $h_m = \frac{1}{2}(h_a + h_b)$  or  $h_a + h_b = 2h_m$ . Also,

$$\text{Area}(ADN) + \text{Area}(BNC) = \frac{1}{2} \times h_a \times DN + \frac{1}{2} h_b \times CN = \frac{1}{2} \times h_a \times DN + \frac{1}{2} h_b \times DN$$

(since  $CN = DN$ )

$$= \frac{1}{2} \times DN \times (h_a + h_b)$$

$$\begin{aligned} \text{Area}(DMC) &= \frac{1}{2} \times h_m \times DC = \frac{1}{2} \times \frac{1}{2} \times (h_a + h_b) \times (DN + NC) \\ &= \frac{1}{2} \times \frac{1}{2} \times (h_a + h_b) \times (2DN) = \frac{1}{2} \times (h_a + h_b) \times DN \end{aligned}$$

Hence  $\text{Area}(ADN) + \text{Area}(BNC) = \text{Area}(DMC)$  or  $S_1 + S_3 + S_6 + S_7 = S_3 + S_4 + S_6$  or  $S_1 + S_7 = S_4$ .

**Answer: (B)**

**Question 13**

We will attempt to find a pattern when  $2^n$  is divided by 7 (because there are 7 days in a week), where  $n$  is a positive integer.

When  $n = 1$ , the remainder is 2.

When  $n = 2$ , the remainder is 4.

When  $n = 3$ , the remainder is 1.

When  $n = 4$ , the remainder is 2.

When  $n = 5$ , the remainder is 4.

When  $n = 6$ , the remainder is 1.

When  $n = 7$ , the remainder is 2.

This shows that  $2^n$  leaves a remainder of 2 when divided by 7 if  $n$  leaves a remainder of 1 when divided by 3. Since 2020 leaves a remainder of 1 when divided by 3,  $2^{2020}$  leaves a remainder of 2 when divided by 7. The question is now equivalent to finding which day is 2 days after Tuesday, which will be **Thursday**.

Answer: **(B)**

**Question 14**

Assume Ellie and Audrey are from Westwood Secondary School, then Tom, Chris and Violet are from Greenwich Secondary school. This contradicts the fact that Tom and Chris are from different schools. Hence, Ellie and Audrey are from Greenwich Secondary School.

Similarly, assume Violet and Chris are Secondary 3 students, then Audrey, Tom and Ellie are Secondary 1 students. This contradicts the fact that Audrey and Tom study at different levels. Hence, Violet and Chris are Secondary 1 students.

To find the Secondary 3 student from Westwood Secondary School, we can eliminate Violet and Chris who are Secondary 1 students, and Ellie and Audrey who are from Greenwich Secondary school. This leaves **Tom** as the only student who fulfils the question's criteria.

Answer: **(D)**

**Question 15**

Note that  $5!$ ,  $6!$ ,  $7!$ ,  $8!$  and  $9!$ , have one consecutive digit '0' at the end since each of them contain exactly one '5' in the product.

Next,  $10!$  to  $11!$  have two consecutive digit '0's at the end since each of them contain exactly two '5's in the product.

We continue listing and notice that  $20!$  to  $21!$  have 4 consecutive digit '0's at the end, but  $25!$  to  $29!$  have 6 consecutive digit '0's at the end. Thus, it is impossible to have 5 zeroes at the end of  $n!$ .

Similarly, we obtain that it is impossible to have 11, 17, 23, 29, 30 or 36 zeroes at the end of  $n!$ .

Thus, there are **7 integers** from 1 to 40 which cannot be the number of consecutive digit "0"s at the end of  $n!$ .

Answer: **(C)**

**Question 16**

Let each space of the bar graph be  $x$  students.

Number of students who answered the question correctly = number of students who chose B =  $7x$ . Hence, a total of  $14x$  points were scored from them.

Number of students who answered the question wrongly = number of students who chose A, C, D or E =  $3x + 5x + 8x + 4x = 20x$ . A total of  $-20x$  points were scored from them.

Total number of points =  $14x - 20x = -6x = -240$ , or  $x = 40$ .

Number of students who answered the question correctly =  $7x = \mathbf{280}$

Answer: **280**

**Question 17**

Notice the figure is made of 2 overlapping 8 x 8 squares.

We first attempt to calculate the number of possible squares that can be formed in each 8 x 8 square:

Number of 1 x 1 squares =  $8^2$

Number of 2 x 2 squares =  $7^2$

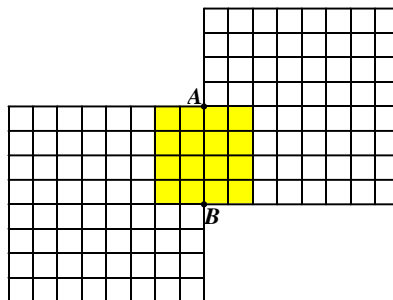
Number of 3 x 3 squares =  $6^2$

...

Number of 8 x 8 squares =  $1^2$

Hence total number of squares in two 8 x 8 squares =  $2 \times (1^2 + 2^2 + 3^2 + \dots + 8^2) = 408$

Finally, notice that there is an overlapping region (shaded in yellow in the diagram below) between the 2 squares, which has a width of 4 small squares. From this overlapping region, all squares formed must pass through the line AB. Hence, number of 2 x 2 squares = 3, number of 3 x 3 squares = 4 and number of 4 x 4 squares = 3.



Total number of squares =  $408 + 3 + 4 + 3 = \mathbf{418}$

Answer: **418**

**Question 18**

Let the time Chloe took to reach the cinema be  $x$  minutes, then distance from home to the cinema is  $100x$  metres.

At 12.10 p.m., 30 minutes after 11.40 a.m., her cousin had walked  $60 \times 30 = 1800$  metres towards the cinema, while Chloe had walked  $100(30 - x - 10) = (2000 - 100x)$  metres towards home. Notice that this was when Chloe and her cousin had met, which means that the total distance Chloe and her cousin covered at 12.10 p.m. is the distance between Chloe's home and the cinema.

$$2000 - 100x + 1800 = 100x$$

$x = 19$  minutes. Hence, the distance is **1900** metres.

Answer: **1900**

**Question 19**

Let  $n^2 + 11n + 52 = x^2$  for some positive integer  $x$ .

We complete the square for the equation  $n^2 + 11n + 52 = x^2$  to get  $\left(n + \frac{11}{2}\right)^2 + \frac{87}{4} = x^2$  or  $\frac{(2n+11)^2}{4} + \frac{87}{4} = x^2$

Multiply the above equation by 4, and we can express  $m = 2n + 11$  to get  $m^2 + 87 = (2x)^2$  or  $(2x + m)(2x - m) = 87$ . Since  $2x + m > 2x - m$ , and  $2x + m$  and  $2x - m$  are positive integer factors of 87,

$2x + m = 29$  and  $2x - m = 3$ , which gives  $m = 13$ , or  $2x + m = 87$  and  $2x - m = 1$ , which gives  $m = 43$ . When  $m = 13$ ,  $n = \frac{13-11}{2} = 1$ . When  $m = 43$ ,  $n = \frac{43-11}{2} = 16$

Hence, the sum of all such  $n$  is  $1+16=$ **17**.

Answer: **17**

**Question 20**

If 7A34B4 is divisible by 36, it must be divisible by 4. By divisibility rule, B4 must be divisible by 4. Since B is a prime,  $B = 2$ .

If 7A34B4 is divisible by 36, it must be divisible by 9. By divisibility rule,  $7 + A + 3 + 4 + 2 + 4$ , or  $20 + A$  must be divisible by 9.

Since A is a one-digit number,  $A =$  **7**

Answer: **7**

**Question 21**

The pattern is as follows:  $a \wedge b = [2(a + b)][3(b - a)]$

where we don't multiply the square brackets, but write these numbers next to each other. E.g.  $4 \wedge 8 = [2(8 + 4)][3(8 - 4)] = 2412$

This is a very common "trick" in SASMO, and must be known by heart!

$$7 \wedge 8 = [2(7 + 8)][3(8 - 7)] = \mathbf{303}$$

Answer: **303**

**Question 22**

In the middle of the manual count, there are  $1400 - 300 - 154 - 194 - 281 - 266 - 9 = 196$  votes which are yet to be counted.

To ensure A wins the post, we consider the worst-case scenario, when all remaining votes are either for A, or for the candidate with the 2<sup>nd</sup> highest vote count, D.

In this scenario, D needs 20 more votes than A (since A is currently leading by 19 votes), from the remaining 196 votes to win, or  $\frac{196+20}{2} = 108$  votes.

Hence A needs a minimum of  $300 + 196 - 108 + 1 = \mathbf{389}$  votes to win.

Answer: **389**

**Question 23**

Let the length of AC be  $5x$  and the length of BC be  $5y$ .

By Pythagoras theorem,

$$EC^2 + BC^2 = (2x)^2 + (5y)^2 = 4x^2 + 25y^2 = BE^2 = 10, \text{ and}$$

$$AC^2 + CD^2 = (5x)^2 + (3y)^2 = 25x^2 + 9y^2 = AD^2 = 16$$

Solving the simultaneous equations, we get  $x^2 = \frac{10}{19}$  and  $y^2 = \frac{6}{19}$

One final application of Pythagoras theorem reveals that  $AB^2 = AC^2 + BC^2 = (5x)^2 + (5y)^2 = 25(x^2 + y^2) = 25 \times \left(\frac{10}{19} + \frac{6}{19}\right) = 25 \times \frac{16}{19} = \frac{400}{19}$

$$57 \times AB^2 = 57 \times \frac{400}{19} = \mathbf{1200}$$

Answer: **1200**

**Question 24**

Let there be

- $c$  roads between Town A and B
- $a$  roads between Town B and C
- $b$  roads between Town C and A

One can go from Town A to Town C directly or via Town B in a total of 33 different ways:

$$b + ca = 33 \quad (1)$$

One can go from Town B to Town C directly or via Town A in a total of 23 different ways:

$$a + bc = 23 \quad (2)$$

Subtracting (2) from (1):

$$33 - 23 = 10 = b + ca - a - bc = b - a + c(a - b) = (c - 1)(a - b)$$

Since  $10 \geq a, b, c \geq 3$ , then  $(c - 1) = 2$  or  $5$ ,  $c = 3$  or  $6$  and  $(a - b) = 5$  or  $2$ .

From (1),

$$\text{if } c = 3 \text{ and } a - b = 5, \text{ then } 33 = b + ca = b + 3 \times (5 + b) = 15 + 4b \Rightarrow b = 4.5$$

However, it is not possible to get 4.5 ways.

$$\text{if } c = 6 \text{ and } a - b = 2, \text{ then } 33 = b + ca = b + 6 \times (2 + b) = 12 + 7b \Rightarrow b = 3 \text{ and } a = 5$$

From Town B to Town A, there are  $c + ab = 6 + 3 \times 5 = 21$  ways.

Answer: **21**



**Question 25**

$S = 5$  because  $S \neq M$  and there can be a carrying over of maximum 1 to  $S$ , so  $S + 1 = 6$ .  $A = 9$  because  $A$  must carry a 1 over to  $S$ , so  $A + 1 \geq 10$ . When  $A = 9$ ,  $E = 0$ .

Substitute  $N = 4$ ,  $M = 6$ ,  $S = 5$ ,  $A = 9$  and  $E = 0$  to get the following:

$$\begin{array}{rcccccc}
 & & & W & I & 4 \\
 + & 5 & 9 & 5 & 6 & O \\
 \hline
 & 6 & 0 & D & 9 & L
 \end{array}$$

We are now left with 5 numbers which  $W$ ,  $I$ ,  $O$ ,  $D$  and  $L$  represent, namely 1,2,3,7 and 8. With these numbers, there are only 2 cases left to consider for  $4 + O = L$ .

Case 1:  $O = 3$  and  $L = 7$ . But this will leave no possible value for  $I$  to satisfy  $I + 6 = 9$  since  $O = 3$ . Hence case 1 is impossible.

Case 2:  $O = 7$  and  $L = 1$ . Then  $I = 2$  for  $1 + I + 6 = 9$ . Finally,  $W = 8$  and  $D = 3$  for  $W + 5 = D + 10$

$$M + E + D + A + L = 6 + 0 + 3 + 9 + 1 = \mathbf{19}$$

Answer: **19**